COVARIANCE MATRIX FORECASTING

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Covariance matrix forecasts of financial asset returns are an important component of current practice in financial risk management [4]. Modelling of the second moments of asset returns has been a major field of study in finance over the last 20 years [1]. Forecasts of volatilities and correlations are of crucial importance in risk management [2].

The goals of this paper are the following: i) to analyze different methods that can be used for covariance matrix forecasting and ii) to present the results of an empirical research performed using real FX data.

In order to understand main techniques of covariance matrix forecasting let's reduce the problem to variance forecasting techniques. One of the main notions used in forecasting is return during one day. Continuously compounded return during day i is defined as (equations below are taken from [3]):

$$u_i = \ln \frac{S_i}{S_{i-1}}$$

where:

S_i is the value of the market variable at the end of day i;

S_{i-1} is the value of the market variable at the end of day i-1.

For the purposes of calculating Value at Risk, variance rate is given by:

$$\sigma^2{}_n = \frac{1}{m}\sum_{i=1}^{m} u^2{}_{n-i}$$

Equation above gives equal weight to all u^2's. Because the recent data is more important, it is appropriate to give more weight to recent data. A model that does this is:

$$\sigma^2{}_n = \sum_{i=1}^{m} \alpha_i u^2{}_{n-1}$$

were α_i is the amount of weight given to the observation i days ago. All α's are positive and the weights must sum to unity.

If to assume that there is a long-run average volatility V given weight γ, the model becomes:

$$\sigma^2{}_n = \gamma V + \sum_{i=1}^{m} \alpha_i u^2{}_{n-1}$$

This is known as ARCH (m) model, first suggested by Engle (1982). The estimate of the variance is based on a long-run average variance and m observations.

If the weights a_i decrease exponentially as we move back through time, we have the exponentially moving average model (EWMA):

$$\sigma^2{}_n = \lambda\sigma^2{}_{n-1} + (1-\lambda)u^2{}_{n-1}$$

where:

λ is a constant between zero and one.

It is worth to note that JP Morgan uses the EMWA model with λ=0.94 for updating daily volatility estimates in its RiskMetrics database.

Bollerslev in 1986 proposed the GARCH (1,1) model, the equation for which is:

$$\sigma^2{}_n = \gamma V + \alpha u^2{}_{n-1} + \beta\sigma^2{}_{n-1}$$

where g is the weight assigned to V, α is the weight assigned to u^2_{n-1}, and β is the weight assigned to s^2_{n-1}. Weights must sum to unity.

Now the question arises which model to choose? In practice, variance exhibits mean reversion.

It may be showed that the GARCH (1,1) model incorporates mean-reversion whereas the EWMA model does not. So from the theoretical point of view GARCH (1,1) must be used where possible.

Covariances between two variables can be updated in a similar way to variance estimates. Covariance estimate between U and V is:

$$\mathrm{cov}_n = \frac{1}{m}\sum_{i=1}^{m} u_{n-1}v_{n-1}$$

An EWMA model for updating the covariance estimate is:

$$\mathrm{cov}_n = \lambda\,\mathrm{cov}_{n-1} + (1-\lambda)u_{n-1}v_{n-1}$$

And similarly, the GARCH (1,1) model for updating a covariance is:

$$\mathrm{cov}_n = \omega + \alpha u_{n-1} v_{n-1} + \beta \,\mathrm{cov}_{n-1}$$

Statistical loss functions are used for evaluation of covariances forecasts. Most popular function is the root mean squared error (RMSE), which is given by [1]:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(u^2{}_i - \sigma^2{}_i)^2}$$

The smaller the quantity the better the forecast.

Daily closing prices on 6 major USD foreign exchanges (AUD, CAD, CHF, EUR, GBP, JPY) from 23-Mar-00 to 8-Jul-02 were used for variance and covariance matrix forecasting.

First of all I tried to determine the time period that produces most accurate forecasting results. Time series of daily closing process of USD/AUD rates were used for variance forecasting using the EWMA model. The results are presented in Table 1.

Table 1. RMSE ($*10^{12}$) for USD/AUD

Days	30	60	90	120	150	180	210	250
Result	27,73	17,94	15,77	15,58	15,41	14,13	147,02	150,04

As it can be seen, the most accuarate forecasts are when 180 days period is used. This result is used further for covariance matric forecasting. Results showed that forecasts and actual covariances matrixes fit to each other rather well.

The following conclusions can be made:

1) Covariance matrix forecasting is a very important issue in up-to-date risk management practice;

2) a great variaty of models for covariance matric forecasting is available for risk manager starting from exponentialy weighted moving average model to various extensions of GARCH models;

3) it can be proved that GARCH (1,1) model incorporates mean-reversion while EWMA does not, so from the theoretical point of view GARCH (1,1) must be used were possible;

4) using daily closing prices on 6 major USD foreign exchanges the performance of EWMA model was tested. Results showed that must accurate forecasts are when the period of 180 days is used and that using this time interval covariance matrix forecasts are accurate enough.

References

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KOVARIACIJŲ MATRICOS PROGNOZAVIMAS

Santrauka

Kovariacijų matrizos prognozavimas yra labia svarbus rizikos valdymo praktikoje.

Šiame pranešime yra pateikiami pagrindiniai kovariacijų matricai prognozuoti naudojami modeliai bei atliekamas eksponentiškai pasverto judančio vidurkio modelio testavimas naudojant šešių pagrindinių užsienio valiutų kursus.

Kovariacijų matricai prognozuoti galima taikyti daug modelių, pradedant eksponentinio svertinio judančio vidurkio modeliu ir baigiant įvairiomis GARH modelio modeifikacijomis.

Pastebėta, kad GARCH (1,1) modelis apima savyje finansų rinkoms būdingą savybę – sugrįžimą prie vidurkio, o EWMA modelis – ne. Taigi, iš teorinės pusės labiau reikėtų taikyti GARCH (1,1) modelį.

Pranešime taip pat pateikiami EWMA modelio testavimo rezultatai, kurie rodo, kad tiksliausios prognozės gaunamos, kai naudojamas 180 dienų prognozavimo periodas ir kad, naudojant šį periodą, kovariacijų matricos prognozės yra pakankamai tikslios.