RISK ADJUSTMENT AND PERFORMANCE MEASUREMENT: SYMMETRICAL VERSUS ASYMMETRICAL MEASURES

Audrius Dzikevičius
Vilnius Gediminas Technical University
Saulėtekio al. 11, LT-10223 Vilnius-40, Lithuania
E-mail: audrius.dzikevicius@takas.lt
Received 31 January 2005; accepted …

Abstract. Risk adjustment of returns and performance measurement is of great interest to financial institutions around the world today, because management board of a certain financial institution wants to know what risks their institution is bearing while achieving a certain level of returns. Separate measures of risk and return are combined in a single ratio through a risk adjustment process and corresponding measures that are analysed in the paper. All risk adjustment measures are divided into two separate classes in the paper according to normality of returns of market variables or portfolios criterion. The analysis revealed the following conclusions: when the normality assumption holds, the generalized Sharpe rule is superior to other risk adjustment measures from the class; when we cannot rely on the normality assumption, then Farinelli-Tibiletti ratio is superior to other asymmetrical risk-adjustment measures because it not only accounts for deviations of financial data from normal distribution, but enables to asses the impact of different preferences of an investor towards profits and/or losses expected.

Keywords: risk, returns, risk-adjustment, risk-adjusted measures, normality, asymmetrical preferences.

1. Introduction

Risk adjustment of returns and performance measurement is of great interest to financial institutions around the world today. Management board of a certain financial institution wants to know what risks their institution is bearing while achieving a certain level of returns. This is important not only because of increasing competition among financial institutions, but also because of the wish of shareholders to move from passive risk measurement or setting of limits to active risk management, i.e. financial institutions must improve their operations by optimizing the relationship between financial results and risks taken.

Risk-adjusted measurement is the tool to determine and manage this sort of relationship. Separate measures of risk and return are combined in a single ratio through a risk adjustment process. Risk adjustment process tries to find a common measure that would enable to compare achieved results and risks taken of different portfolios of securities, structural units or even companies. These risk adjustment measures can help to solve a number of practical problems, such as [1]:

- Business valuation,
- Setting of financial goals and measurement the level of attainment of financial results,
- Development and implementation of reward and motivation systems,
- Making decisions of capital allocation, etc.

Although several different risk adjustment measures are known and used in practice today, there is no unanimous decision regarding their applicability to tackle problems of today. Further more, in the scientific literature there is no established and used in the same way terminology: different risk adjustment measures have the same name or different authors name the same measures differently. Despite that, all of them compare achieved financial results with risks taken.

J. L. Treynor [2], W. F. Sharpe [3] and M. Jensen [4] were among the pioneers of this research field. Classical risk adjustment measures were reviewed and summarized by F. K. Reilly and K. C. Brown [5] and C. S. Pedersen et al. [6]. K. Dowd [7, 8] also contributed to the research field considerably.

Because returns of certain financial instruments may not be fitted to normal distribution, a lot of effort was devoted towards creation of alternative risk adjustment measures recently (see, for example, 9 - 14].

The aim of this paper is to compare different risk adjustment measures divided into two separate classes –

measures based on normality assumption and those without such assumption, to discuss their advantages, disadvantages and specific features of application, and also to try to reveal the risk adjustment measures from both classes that overcome competitors.

The goals of the paper are the following:

1) To present the concept of risk adjustment,
2) To perform the comparative analysis of risk adjustment measures assuming normality of returns, reveal their advantages, disadvantages and specific features of application,
3) To perform the comparative analysis of risk adjustment measures allowing for non-normality of returns, reveal their advantages, disadvantages and specific features of application

The following research methods were used in the paper: literature analysis, logic and meta-analysis.

The paper should motivate other researches into the research and improvement of modern risk management tools. This study may be useful to commercial banks and other financial companies that are engaged in the trading activities and development and the implementation of risk adjustment procedures.

2. The concept of risk adjustment

Management of financial institutions and their shareholders seek to see real picture of achieved financial results, because it is important what risks bearing were or will be achieved certain financial results. Financial results and risks taken are being combined through the concept of risk adjustment.

Risk-adjusted measurement may have two aspects [7]:

1. Measurements of alternative investment opportunities before the decisions to invest are made. In which way and what does an investment portfolio manager chose – investment with high expected returns, but also risky, or investment with not so high returns, but which also is rather safe? The answer to this question may help to find measures that link together expected returns with risks to be taken.
2. Measurement of investment returns after the decisions to invest are already made, when the results of the decisions are already clear. In this case one should compare, for example, two different dealers: the first of them achieved high returns but took high risks, while the second one achieved moderate returns on investment, but took nearly no risks on funds of a financial institution. Also there may arise the need to assess not only different dealers, but also investment performance of a certain structural unit or the quality of management of different portfolios of securities.

So from what was said above, one can say that risk-adjusted measurement may have the number of different possible uses starting from the measurement of alternative or already made investments or even companies, setting of financial goals and the measurement of the level of their attainment, development and implementation of reward and motivation systems, making capital allocation decisions, etc.

Example in Fig 1 and Table 1 illustrates the process of risk adjustment. Imagine we have a number of traders, *A* to *E*, who generate the risk-return combinations shown in Fig 1 below. Trader *E* makes the highest return, but also takes more risk than the other traders. On the other side, trader *A* makes the lowest return, but also takes less risk than the other traders. If we rank traders by their returns alone, we will rank *E* first, followed by *D*, *B*, *C*, and *A*. On the other hand, if we rank traders by their risks alone, we will rank *A* first, followed by *B*, *C*, *D*, and *E*. So we have obviously very different rankings. The first ranking gives too much stress on returns, and the second – too much stress on risk. If we want to account for both returns and risk in a single ranking, we will rank *B* first, followed by *D*, *E*, *C*, and finally *A*. So trader *B* achieved the best results according to his risk-adjusted return, and trader *A* achieved the worst result.

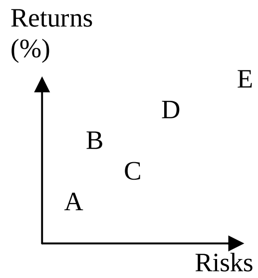

Fig 1. Illustration of risk-adjusted measurement

Table 1. Ranking of dealers

According to returns	According to risks	According to risk-adjusted returns
E	A	B
D	B	D
B	C	E
C	D	C
A	E	A

Risk adjustment may be carried out in a number of different ways. Each risk adjustment measure has its own advantages and disadvantages. Further in the next section, different risk adjustment measures are shortly described and their advantages and disadvantages revealed.

3. Risk adjustment under normality assumption

Until the 1960s portfolio performance management was measured according to generated returns only. The concept of risk was known, but no one knew how to measure it quantitatively. Modern portfolio theory showed investors how risk may be quantified through the standard deviation of returns. Despite that, at that time there was no any quantitative measure aggregating risks and returns, these factors were analyzed separately, i.e. investors grouped investments into similar risk classes according to the standard deviations of returns and then returns of alternative investments in certain risk classes were measured [5].

Classical risk adjustment measures are based on the Capital Asset Pricing Model (CAPM) which proposes that it is worth to invest if expected returns (R_E) of this investment exceed required returns, i.e.:

$$R_E = \text{RFR} + \beta_i (R_m - \text{RFR}), \qquad (1)$$

where:

RFR denotes a risk-free rate;

R_m denotes expected returns on the market portfolio of Risky assets;

β_i denotes beta of *i* risky asset.

CAPM has a number of relevant problems [15, 16], but the most important one related to the object of the paper is that this model does not assess the impact of investment on a certain portfolio under consideration. Risk and return of a prospective investment are compared with hypothetical market portfolio that we should have according to CAPM but in real world no one has such portfolio and, besides, portfolios managed by different portfolio managers differ substantially.

CAPM is based on the assumption that returns on assets are distributed under normal probability distribution.

The normal probability distribution has the following symmetrical probability density function [17]:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \quad (2)$$

where:

μ denotes a mean of the normal random variable;

σ^2 denotes a variance of the normal random variable.

When μ is equal to 0 and σ^2 is equal to 1, we have standard normal variable (see Fig 2).

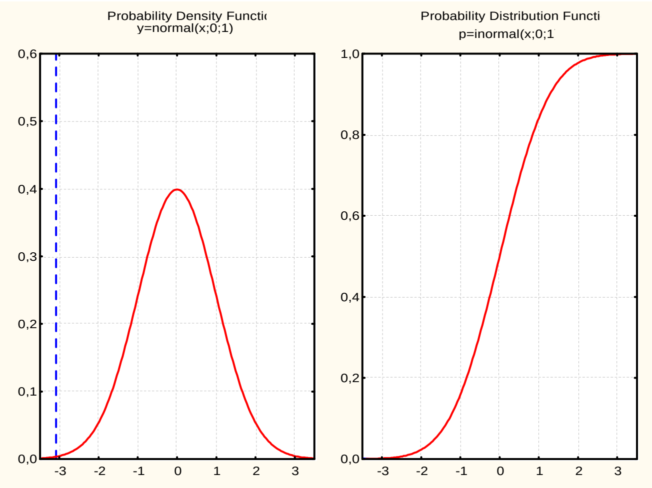

Fig 2. Standard normal variable

Normality has a number of attractive mathematical features that make calculations easier so many theories in economics assume normality of variables. We will discuss further in the section the ratios based on CAPM and respectively assume normality of returns.

Treynor ratio. J. L. Treynor developed the very first measure of portfolio performance that included risk and returns in 1965 [2]. He postulated two components of risk:

1) Risk produced by general market fluctuations, and
2) Risk resulting from unique fluctuations in the portfolio of securities.

In order to identify the risk produced by general market fluctuations he introduced *the characteristic line* which defines the relationship between the rates of return for a portfolio over time and the rates of return for an appropriate market portfolio over the same time period. The slope of the characteristic line measures the relative volatility of the portfolio returns in relation to the returns for the aggregate market. This slope is also known as portfolio beta coefficient. A higher slope (beta) characterizes a portfolio that is more sensitive to market returns and that has greater market risk.

Deviations from the characteristic line indicate unique portfolio returns relative to aggregate market returns. These deviations result due to different returns of individual securities in the portfolio. Such differences would cancel out in the fully diversified portfolio.

J. L. Treynor showed that a rational risk-averse investor would be willing to choose portfolio opportunity lines with higher slopes, because the lines of higher slopes help investors to achieve higher indifference curves. The slope *T* of portfolio possibility line is equal to [2]:

$$T = \frac{R_i - RFR}{\beta_i}, \quad (3)$$

where:

R_i denotes the average rate of return for portfolio *i* during a specified time period,

RFR denotes the average rate of return on a risk-free investment during the same time period,

β_i denotes the slope of characteristic line during that time period (the portfolio relative volatility).

Larger *T* value indicates larger slope and better portfolio for all investors regardless of their risk preferences. Because the numerator of this ratio is risk premium and the denominator is the measure of risk, the total expression indicates the portfolio risk premium return per unit of risk. All risk-averse investors will try to maximize this value. Beta indicates systemic risk and says nothing about the diversification of the portfolio. So this measure assumes a completely diversified portfolio.

Comparing a portfolio *T* value to a similar value of an aggregate market portfolio indicates whether the portfolio would plot above the security market line. T_m value for an aggregate market portfolio is calculated as follows:

$$T_m = \frac{R_m - RFR}{\beta_m}. \quad (4)$$

In this expression, β_m is equal to 1 (the market beta) and indicates the slope of the security market line. Therefore, a portfolio with higher *T* value than *T* value of an aggregate market portfolio plots above the security market line, indicating better risk-adjusted financial results.

Treynor ratio was the very first step towards risk-adjusted measurement. This ratio was the first to combine returns of a portfolio of securities and the risk of an aggregate market. The adjustment of returns according to an aggregate market risk may be correct if and only if the portfolio of securities under consideration is fully diversified, however, in practice this situation is more abstract than real.

Sharpe ratio. W. F. Sharpe used this measure to evaluate the performance of mutual funds in 1966 [3]. Sharpe ratio is similar to Treynor ratio; however, it seeks to measure the total risk of portfolio by including the standard deviation of returns, not systemic risk expressed by beta. This measure indicates the risk premium return earned per unit of total risk.

Sharpe ratio, *SR*, is calculated as follows [5]:

$$SR_i = \frac{R_i - RFR}{\sigma_i}, \quad (5)$$

where:

R_i denotes the average rate of return for portfolio *i* during a specified time period,

RFR denotes the average rate of return on risk-free assets during the same time period,

σ_i denotes the standard deviation of the rate of return for portfolio *i* during the time period.

Later W. F. Sharpe [18] presented the other version of Sharpe ratio that was called as *traditional Sharpe ratio* [7]. Suppose we have a portfolio, *i*, with a return R_i. We also observe a benchmark portfolio, *b*, with a return R_b. Let *d* be the differential return $R_i – R_b$. Then traditional Sharpe ratio is calculated as follows:

$$SR_i = \frac{R_i - R_b}{\sigma_d} = \frac{d}{\sigma_d}, \quad (6)$$

where σ_d is the standard deviation of *d*.

This ratio indicates differential return per unit of risk. The traditional Sharpe ratio in one measure captures not only risk, but also returns. A rising return differential or a falling standard deviation increases the traditional Sharpe ratio, and, conversely, a falling return differential or a rising standard deviation decreases the traditional Sharpe ratio. Hence, comparing or choosing between two investment alternatives or alternative portfolios, we choose those with higher Sharpe ratios.

It is important to have in mind, that the traditional Sharpe ratio gives us sufficient information to make decisions, when the returns of alternative investments or structural units are not correlated with the rest of the financial institution‘s portfolio [8].

In the equation of the traditional Sharpe ratio, the standard deviation of portfolio returns over the specified time period stands for risk measure, thus this ratio accounts for both, returns and the level of diversification of a portfolio of securities. Consequently, this measure is much more informative than Treynor ratio. In a fully diversified portfolio of securities case both ratios would be the same because the standard deviation of fully diversified portfolio of securities is equal to the systematic standard deviation. In a poorly diversified securities portfolio case, Treynor ratio would be higher that traditional Sharpe ratio.

The main disadvantage of traditional Sharpe ratio is the fact that the ratio is correct if and only if candidate positions to the portfolio are not correlated with the existing portfolio. If this assumption holds, then, while comparing alternative investments, we choose that with the highest Sharpe ratio. If this assumption does not hold, while comparing alternative investments, it is possible to come to wrong conclusions. For example, let us assume that traditional Sharpe ratio of the investment *A* is lower than that of the investment *B*, returns of the investment *A* negatively correlates with returns of the existing portfolio, returns of the investment *B* positively correlates with returns of the existing portfolio. Then the purchase of asset *A* would reduce portfolio risk, while the purchase of *B* would increase it, and it is possible that we would prefer *A* over *B* if we took these correlation effects into account.

Jensen's alpha. M. C. Jensen applied the measure that today is called Jensen's alpha for a measurement of the performance of mutual funds in 1968 [4].

The Capital Asset Pricing Model calculates the expected one-period return on an asset or portfolio in the following way:

$$E(R_i) = RFR + \beta_i[E(R_m) - RFR], \quad (7)$$

where:

E(R$_i$) denotes the expected return on security or portfolio *i*,

RFR denotes the one-period risk-free interest rate,

β_i denotes the systematic risk (beta) for security or portfolio,

E(R$_m$) denotes the expected return on the market portfolio of risky asset.

If the expectations in the above equation are expressed in terms of realized rates of return, we will have the following expression:

$$R_{it} = RFR_t + \beta_i[E(R_{mt}) - RFR_t] + U_{it}\,. \quad (8)$$

One can see in the equation (8) that realized returns of security or portfolio over a specified period is a linear function of retunes of a certain investment over a specified period plus risk premium that depends on systematic risk of certain security or portfolio plus a random error term.

A subtraction of risk-free rate from the both sides of the equation gives the following:

$$R_{it} - RFR_t = \beta_i[E(R_{mt}) - RFR_t] + U_{it}\,. \quad (9)$$

Equation (9) states that the risk premium of portfolio *i* is equal to β_i multiplied by market risk premium plus a random error term. Hence, if this equation holds, the regression intercept α must be equal to zero. To measure superior investment returns, one must allow for a non zero intercept α, that will be positive when the manager of securities portfolio achieves higher returns than aggregate market, and will be negative, when the manager of securities portfolio achieves lower returns than aggregate market:

$$R_{it} - RFR_t = \alpha_i + \beta_i[E(R_{mt}) - RFR_t] + U_{it}\,. \quad (10)$$

Jensen's alpha is based on the same principles as Treynor ratio or traditional Sharpe ratio, and Jensen's alpha has the same disadvantages that are characteristic to those of Treynor ratio and the traditional Sharpe ratio: they are subject to generic weaknesses of CAPM and they apply to mean-variance world.

The information ratio. The information ratio indicates a portfolio average return in excess of a benchmark portfolio over the some time period divided by the standard deviation of this excess return [19]:

$$IR_i = \frac{R_i - R_b}{\sigma_d}, \quad (11)$$

where:

IR_i denotes the information ratio for portfolio *i*,

R_i denotes the average return for portfolio *i* during the specified time period,

R_b denotes the average return for the benchmark portfolio during the same time period,

σ_d denotes the standard deviation of the excess return during the same time period.

It is not difficult to notice that this measure is the same as traditional Sharpe ratio, just called the information ratio.

K. Dowd presented different version of the information ratio [7]:

$$IR_i = \frac{R_i}{\sigma_d}. \quad (12)$$

W. F. Sharpe [18] shows that the information ratio may lead to misleading decisions. This can be demonstrated by the example. Let us assume that an investor has a choice of two alternative funds, *X* and *Y*. Fund *X* has an expected return of 5% and a standard deviation of 10%, and fund *Y* has an expected return of 8% and a standard deviation of 20%. Therefore, fund *X* has an information ratio of 0.5, and fund *Y* one of 0.4, and so the information ratio criterion would lead us to prefer *X* to *Y*. Now suppose that the risk-free interest rate is 3%, therefore, fund *X* has the traditional Sharpe ratio of 0.2, and fund *Y* one of 0.25, and so according to the traditional Sharpe ratio criterion would lead us to prefer Y over X, and it is easy to show that this is correct choice. The information ratio is misleading because it does not account for the cost of funds.

Treynor-Black ratio. The other alternative risk-adjustment measure is Treynor-Black ratio that, in the end, is the same traditional Sharpe ratio, only squared [7]:

$$TBR_i = \left(\frac{R_i - R_b}{\sigma_d}\right)^2. \quad (13)$$

Squaring obscures information, hence, it may be misleading. For instance, an investor has a choice of two alternative funds, *X* and *Y*. Fund *X* has an expected return of 2% and a standard deviation of 5%, and fund *Y* has an expected negative return of -2% and a standard deviation of 5%, risk-free interest rate is equal to 3%. Therefore, fund *X* has Treynor-Black ratio of 0.04, and fund *Y* one of 0.04 also, and so according to this criterion both funds are the same, but it is clear that this is not true.

The generalized Sharpe rule. The generalized Sharpe rule was developed as a response to the following main disadvantage of the traditional Sharpe ratio: a restriction regarding correlation of an asset under consideration with the portfolio. Suppose, we a have a portfolio and are considering buying an additional asset. In order to overcome this correlation problem inherent for the traditional Sharpe ratio all we need to do is construct two Sharpe ratios, one for the old portfolio taken as a whole, and one for the new portfolio or the portfolio we would have if we add a new asset to the old portfolio. Denote the old Sharpe ratio by SR^{old}, and the new one by SR^{new}, and then we would make a decision to complement the existing portfolio with a new asset if and only the following inequality holds [8]:

$$\text{Buy the new asset, if and only if } SR^{new} \geq SR^{old}. \quad (14)$$

We complement the old portfolio with the new asset if and only if the new portfolio has a Sharpe ratio greater than that of the old portfolio.

So the generalized Sharpe rule has no main disadvantage of the traditional Sharpe ratio, that virtually hinders its application possibilities, i.e. it is not based on the assumption that returns of the candidate portfolio positions does not correlate with the existing portfolio. Since two different traditional Sharpe ratios are computed together and compared with each other, this rule avoids the above-mentioned disadvantage and, in my opinion, is most reasonable of all and has wide possibilities for practical applications that are beyond the limits of this article. This is an approach that really may be of help in maximizing shareholders value of companies assuming normality.

4. Risk adjustment under non-normality assumption

Leaving behind the assumption of normality in return distributions, the classical risk-reward ratios based on CAPM become a questionable tools, because they equally take into account both positive and negative changes in returns.

Significant deviations from normality have been demonstrated for emerging markets and portfolios with derivatives [6]. Because returns of certain financial instruments may not be fitted to normal distribution, a lot of effort was devoted towards creation of alternative risk adjustment measures recently (see, for example, 9–14]. These alternative measures allow us to compare to the benchmark returns distributed not normally.

The classical Sharpe ratio equally accounts for "good" volatility (above the benchmark) and "bad" volatility. This ratio is fully compatible with normally distributed returns. By relaxing this artificial assumption of normality we arrive into an asymmetrical world where "good" and "bad" volatility may differ strongly. And, moreover, if we to model asymmetrical preferences towards "good" and "bad" volatility from the benchmark or we want to account for "small" and "large" deviations from the benchmark, we need asymmetrical risk adjustment measures.

Below we present some examples of alternative risk adjustment measures.

The Minimax ratio. Mathematical expression of this ratio is the following [20]:

$$MM_i = \frac{R_i - R_b}{R_b - \min r_t}, \quad (15)$$

where $1 \leq t \leq T$ and r_t is the vector of returns at time t.

This ratio accounts for asymmetrical returns capturing in denominator minimal value of returns observed over period under consideration. It may be demonstrated that the larger the losses would be recorded, the lower the ratio would be calculated.

Sortino-Satchel ratio. Mathematical expression of this ratio is the following [9, 11]:

$$SS_i = \frac{R_i - R_b}{\sqrt[q]{\frac{1}{T}\sum_{i=1}^{T}(r_i - \frac{R_b}{2})_-^{\ q}}}, \quad (16)$$

where q denotes the left order of the distribution density of returns.

The larger is q, the more stress we put on negative outcomes in the left tail. Here in the denominator we concentrate on the negative events only. This allows to account for investor preferences towards risk tolerance and asymmetrical behaviour of returns.

Farinelli-Tibiletti ratio. Mathematical expression of this ratio is the following [14]:

$$\Phi_b^{\ p,q}(X) = \frac{\sqrt[p]{\frac{1}{T}\sum_{i=1}^{T}(r_i - R_b)_+^{\ p}}}{\sqrt[q]{\frac{1}{T}\sum_{i=1}^{T}(r_i - R_b)_-^{\ q}}}, \quad (17)$$

where:

$(r_i - R_b)_+^{\ p} = (\max(r_i - R_b, 0))^p$;

$(r_i - R_b)_-^{\ q} = (\max(R_b - r_i, 0))^q$;

$p, q > 0$, p and q are respectively right and left orders of risk adjustment measure.

Farinelli-Tibiletti ratio is nothing but the ratio between the favourable events and the unfavourable ones.

When the benchmark, b, is fixed, the higher the ratio, the more preferable is the risky asset.

The orders of the ratio p ir q are chosen following the following reasoning. The magnitude of order q depends on the relevance given to the outcomes on the left tale of the probability distribution. For a given benchmark b, the left order q reflects agent's feeling about the relative consequences of falling below b. If the main target of the investor is to hit the target without particular regard to the amount, it is advisable to choose small magnitude of q. If small deviations below the benchmark b are relatively harmless when compared to large deviations, then it is advisable to choose large magnitude of q. Analogous reasoning applies when choosing the magnitude of right order p.

Let $p = q = 1$, then Farinelli-Tibiletti ratio reduces to the following expression [14]:

$$\Phi_b^{\ 1,1}(X) := \frac{E(r - R_b)_+}{E(r - R_b)_-} \quad (18)$$

This ratio equally accounts for negative and positive events (losses and gains).

The meaning of Farinelli-Tibiletti ratio we will illustrate in examples. Let us consider the following data (Table 2).

Table 2. Farinelli-Tibiletti ratio: illustrative data

Weights	0.33	0.33	0.33
Returns of asset X	3	4	11
Returns of asset Y	1	8	9
Differential return of asset X ($R_b = 6$)	-3	-2	+5
Differential return of asset Y ($R_b = 6$)	-5	+2	+3

Both assets, X and Y, have the same differential return, average and variance, their Farinelli-Tibiletti ratios are also the same and equal to 1, but we see that characteristics of risk and return differs. Asset X demonstrates moderate losses (-3, -2) and on the other hand the chance of one high stake (+5), while asset Y displays moderate favourable returns (+2, +3) and a possibility of a large loss (-5).

Suppose that an investor strives to escape huge losses, then his $p < q$. If $p = 1$ and $q = 2$, then with reference to (17) we get the following: Farinelli-Tibiletti ratio of asset X is equal to 0.8, and the one of asset Y is equal to 0.57, so X should be preferred to Y.

Suppose that an investor cares no about losses but rather seeks to earn high profits. If $p > q$, $p = 2$ and $q = 1$, then with reference to (17) we get the following: Farinelli-Tibiletti ratio of asset X is equal to 1.73, and the one of asset Y is equal to 1.25, so X should be preferred to Y.

Suppose that an investor does not care about large losses, but it is important to him to overcome the benchmark, then his $p < q$. If $p = 0.5$ and $q = 1$, then with reference to (17) we get the following: Farinelli-Tibiletti ratio of asset X is equal to 0.33, and the one of asset Y is equal to 0.66, so Y should be proffered to Y.

Farinelli-Tibiletti ratio is also sensitive to the benchmark – the ratio decreases as the benchmark asset R_b increases. Let us consider another example (Table 3).

Table 3. Farinelli-Tibiletti ratio: illustrative data II

Weights	0.33	0.33	0.33
Returns of asset X	3	4	11
Returns of asset Y	1	8	9
Differential return of asset X ($R_b = 4$)	-1	0	+7
Differential return of asset Y ($R_b = 4$)	-3	+4	+5

In this case the return on benchmark R_b is equal to 4, Farinelli-Tibilettti ratios of both assets are not equal to 1, when $p = q = 1$: Farinelli-Tibiletti ratio of asset X is equal to 7, and the one of asset Y is equal to 3, so an investor will prefer X over Y.

Assets X and Y have the same traditional Sharpe ratios and Farinelli-Tibiletti ratios differ. So the latter is more skewness-sensitive than Sharpe ratio.

To conclude, when we cannot rely on the normality assumption, Farinelli-Tibiletti ratio is the most developed from asymmetrical risk-adjustment ratios because it captures not only skewed and heavy tailed distributions, but asymmetrical preferences of an investor as well.

5. Conclusions

After the analysis of two classes of risk adjustment measures the following conclusions can be made:

1. While measuring returns of alternative investments *ex ante* or investments *ex post*, it is important to account not only for expected or achieved returns, but also for expected or taken level of risk. This sort of analysis may be carried out using risk adjustment measures.
2. Classical risk adjustment measures are based on the Capital Asset Pricing Model which assumes the normality of returns.
3. When the normality assumption holds, the generalized Sharpe rule is preferable in assessment of risk-adjusted returns or performance measurement.
4. When we cannot rely on the normality assumption, Farinelli-Tibiletti ratio is the most developed from asymmetrical risk-adjustment ratios because it captures not only skewed and heavy tailed distributions, but asymmetrical preferences of an investor as well.

References

1. Studer, G. Maximum Loss for Measurement of Market Risk, Doctoral Dissertation, Swiss Federal Institute of Technology Zurich, 1997.
2. Treynor, J. L. How to Rate Management of Investment Funds // *Harvard Business Review* 43 (1), 1965, p. 63–75.
3. Sharpe, W. F. Mutual Fund Performance. *Journal of Business* 39, Supplement on Security Prices, 1966, p. 119–138.
4. Jensen, M. The Performance of Mutual Funds in the Period 1945-1964. *Journal of Finance* 23 (2), 1968, p. 389–416.
5. Reilly, F. K.; Brown, K. C. Investment Analysis and Portfolio Management, 6th ed., 1999.
6. Pedersen, C. S.; Rudholm-Alfin, T. Selecting a Risk-Adjusted Shareholder Performance Measure, Working paper, 2003.
7. Dowd, K. Beyond Value at Risk: The New Science of Risk Management. John Wiley & Sons, Chichester, 1999. 274 p.
8. Dowd, K. Adjusting for Risk: an Improved Sharpe Ratio. *International Review of Economics & Finance* 9, 2000, p. 209–222.
9. Sortino, F. Upside-Potential Ratios Vary by Investment Style. *Pensions and Investments* 28, 2000, p. 30–35.
10. Pedersen, C. S.; Satchell, S. E. On the Foundation of Performance Measures Under Asymmetric Returns. Technical Report, Cambridge University, 2002.
11. Sortino, F.; Kuan, B. The Upside Potential Strategy: A Paradigm Shift in Performance Measurement. *Senior Consultant*, Volume 6 (12), 2003.
12. Sortino, F.; van der Meer, R.; Plantinga, A.; Forsey H. The Upside Potential Ratio: What Are We Trying to Measure? *Senior Consultant*, Volume 6 (11), 2003.
13. Goetzmann, W.; Ingersol, J.; Spiegel, M.; Welch, I. Sharpening Sharpe Ratios. Technical Report, Yale School of Management, 2002.
14. Farinelli, L.; Tibiletti, L. Sharpe Thinking with Asymmetrical Preferences. Technical report, University of Torino, 2003.
15. Roll, R. A Critique of the Asset Pricing Theory‘s Tests. *Journal of Financial Economics* 4, 1977, p. 129–176.
16. Frankfuerter, G. M. The Rise and the Fall of the CAPM Empire: a Review on Emerging Capital Markets. *Financial Markets and Instruments* 5 (4). Recent Developments in Financial Economics: Selected Surveys of the Literature, 1995, p. 104–127.
17. Sakalauskas, V. Statistics with Statistica (Statistika su Statistika). Vilnius: Margi raštai, 1998. 227 p.
18. Sharpe, W. F. The Sharpe Ratio. *Journal of Portfolio Management* 21 (1), 1994, p. 49–58.
19. Goodwin, T. H. The Information Ratio // *Financial Analysts Journal* 54 (4), 1998, p. 34–43.

VERTINIMAS, KOREGUOTAS PAGAL RIZIKĄ: SIMETRINĖS IR ASIMETRINĖS METODIKOS

Audrius Dzikevičius
Vilniaus Gedimino technikos universitetas
Saulėtekio al. 11, LT-10223 Vilnius-40, Lietuva
E-mail: audrius.dzikevicius@takas.lt
Gauta 2005 01 31; priimta …

Santrauka. Šiandien finansų institucijos visame pasaulyje labai domisi vertinimu, koreguotu pagal riziką, nes jų vadovybė bei akcininkai nori žinoti realius planuojamų pasiekti ar jau pasiektų finansinių rezultatų vertinimus, kadangi yra svarbu, kokią riziką prisiimant bus arba buvo pasiekta vienokių ar kitokių rezultatų. Finansinius rezultatus ir prisiimamą riziką į vieną rodiklį bando susieti vertinimo, koreguoto pagal riziką, koncepcija bei atitinkamos metodikos, kurios ir yra straipsnyje nagrinėjamos. Vertinimo, koreguoto pagal riziką, metodikos straipsnyje suskirstytos į dvi skirtingas klases pagal tai, ar jie paremti prielaida, kad tiriamų rinkos kintamųjų ar portfelių pelningumai yra pasiskirstę pagal normalųjį pasiskirstymo dėsnį, ar ne. Atlikta analizė leido padaryti tokias išvadas: kai prielaida dėl normaliojo skirstinio galioja, apibendrinta *Sharpe* metodika yra pranašesnė už kitas šios klasės vertinimo, koreguoto pagal riziką, metodikas; kai negalima taikyti prielaidos dėl normaliojo skirstinio, tuomet *Farinelli-Tibiletti* koeficientas yra pranašesnis už kitus asimetrinius vertinimo, koreguoto pagal riziką, rodiklius, nes jis įvertina ne tik tiriamų finansinių duomenų nuokrypas nuo normaliojo skirstinio, bet ir leidžia atsižvelgti į skirtingas investuotojo preferencijas laukiamo pelno ir/ar nuostolio atžvilgiu.

Raktažodžiai: rizika, pelningumas, vertinimas, koreguotas pagal riziką, vertinimo, koreguoto pagal riziką, metodikos, normalumas, asimetrinės preferencijos.

Apie autorių anglų kalba

Audrius Dzikevičius. Doctoral student of Department of Economics and Management of Enterprises at the Vilnius Gediminas Technical University. Address: Saulėtekio av. 11, LT-2054 Vilnius. Tel.: (8 5) 23 93 462, e-mail: Audrius.Dzikevicius@takas.lt. Research interests: market risk measuring and management, portfolio risk management.